ORC 64-8 (RR)
4 April 1964

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This research was supported in part by the Pacific Southwest Forest and Range Experiment Station of the U.S. Forest Service Under Supplement No. 40 of Master Contract A5fs-16565 with the University of California.

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1. INTRODUCTION

A. The Initial Attack Problem — Least Total Cost Objective

The purpose of this study is to analyze the initial attack on wildland fires and to develop decision rules for the related problems of manning, transportation, equipment selection, etc. It has long been recognized that a strong and fast initial attack will prevent most fires from causing substantial dollar losses in either suppression or damage costs, but the questions of "how soon" and "how much" manpower and equipment should be committed to a fire on the initial attack have concerned forest personnel for many years. The response "everything you have available, as soon as possible" is not satisfactory since it begs the more fundamental questions of how much you should have available and the economic implications of "as soon as possible."

The objective to be considered here is that of minimum total cost of suppression plus damages. The interdependencies of area burned and size and strength of initial attack must be recognized, as planning for one cannot be done without consideration of the other. Keeping total area burned to a minimum is an alternative objective, but the suppression costs may be prohibitively high; on the other hand, if no suppression activity is undertaken, the costs of suppression will be nonexistent and the fire damages incurred will be intolerable. Thus, there must be an optimal (least cost) point somewhere in between, and this study will be directed to finding this point.

It may be argued that since firefighting on the National Forests is a public service, the least cost objective is not realistic because a dollar saved in damages is not the same thing as a dollar spent for current suppression activity. However, at least two points may be made to contradict this

argument. First, since an almost unlimited fund of emergency financing becomes available when a fire breaks out of control, it appears that the public is willing to spend unconstrained amounts of current dollars to save future ones. No one seriously considers allowing a large fire to continue burning for lack of finances, so the public should also be prepared to spend current dollars to <u>keep</u> fires from becoming large in the first place. The second point is that since the lands being thus protected belong to the public, loss of them is a real dollar cost to the public. Therefore, it may well be justifiable to spend current public dollars in an effort to operate at minimum total cost levels.

It is also an accepted principle of wildland firefighting that there is some speed-strength tradeoff in initial attack strategies; that is, the quicker you can get to a fire the less men and equipment you will need to control it. In broad terms, the approach to be taken in this study will be that of accepting some time delay between detection of a fire and the start of suppression activity (the attack time) and determining the optimal size suppression force to dispatch to the fire. We shall also examine the possibilities of reducing the attack time to see how this affects both the size of the suppression forces needed and the final size of the fire.

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B. Costs of Fire

If we wish to minimize the total cost of a fire, we must define precisely which costs we intend to consider. We shall consider five types of cost which can be incurred in suppressing a given fire:

1) A fixed cost, associated with maintaining the fire suppression organization and setting it in action at the time of a fire, denoted by C_F (in dollars). This cost, although fixed for a particular fire, will be a function of the operating level of the suppression organization. When the multiple fire situation is considered over a fire season (cf. Section III), it will be treated as another variable cost.

2) A cost which is proportional to the size of the suppression force employed against the fire. This cost will include such items as transportation costs and other "one-shot" logistic support costs, and will be denoted by C_S (in dollars per unit of suppression force).

3) A cost proportional to the total time the fire is burning out of control, which would represent "alert" costs, or "state-of-readiness" costs, denoted by C_T (in dollars per unit of time).

4) A cost measuring the value of property destroyed by the fire, proportional to the final size of the fire, denoted by C_B (in dollars per acre).

5) A cost which is proportional to the total length of time the suppression forces spend controlling the fire. The unit cost, C_X (in dollars per unit of suppression force - unit of time), includes the wages of fire-fighters, the operating costs of equipment, and various expendable and continuing support items.

Various other costs with these dimensions can be incorporated into the parameters just described. For instance, the cost of mop-up operations is probably proportional to the area burned, but not to the size of the force

which suppressed the fire or to the length of time required to obtain control; and could thus be included in C_B .

For the present, we will assume that all the above costs are linear and additive, so that we may write an expression for the total cost of a wildland fire as:

(1.1) Total Cost = C_F + C_S(units of suppression force) + C_T (time required to control the fire) + C_B (size of area burned) + C_X (number of units of suppression force times the time required to control the fire) .

Note that if we send a large suppression force then the second term in (1.1) is dominant, while if the force is small then the third and fourth terms will become more important. Finding the intermediate point of operation which will reduce the total cost to a minimum value — that is, a size suppression force that will make (1.1) minimum, will be our objective.

To fix ideas, in what follows, we shall consider a "man" as the unit of suppression force, but it could be any size fighting unit: a bulldozer, an aerial tanker, a "standard crew," etc. The effect of these men is assumed to be linear; that is, we obtain twice as much effect with twice as many men, and this effect is assumed to be constant throughout the life of the fire; that is, there are no fatigue effects.

C. Difficulties and Advantages in Obtaining Least Cost Solutions

There are several difficulties encountered in a least cost analysis of the wildland fire problem. This study will indicate that these difficulties are not insurmountable by suggesting some possible ways to overcome some of them, and by focussing attention on the ones remaining. Also, it is hoped that this study will serve to encourage others to devote their efforts to the

problem of wildland fire.

The first and undoubtedly the most important difficulty is the lack of quantitative information on predicting the spread of freeburning wildland fire and the effect of various types of suppression action thereon. It is necessary to answer questions of the form "what would have happened if...". In order to do this, models of fire spread as a function of time must be constructed to describe the consequences of, say, extending or reducing the detection time or the attack time. For example, the use of helicopters on most or all fires would be expensive, but if the dollar value of the damages thus prevented exceeded the additional costs, it would be worthwhile.

Further, it is necessary to describe quantitatively the effects of suppression activity on a fire. Presumably the costs of the alternative suppression actions will be known or can be estimated, but predictions must be made of the effectiveness of the alternatives before selecting the best one. Predictions on both fire growth and suppression effectiveness will be based on a large number of physical variables, such as fuel type, wind, slope, moisture content of the fuel, etc., and procedures must be found to assess the influence of these factors. The various fire danger rating systems are designed to help with predicting rates of fire spread, but little or no quantitative information is available for describing the effect of suppression action.

Another major difficulty encountered, solutions to which are outside the scope of this study, is the question of resource valuations, or values at stake. Where necessary in this study to assign actual numbers for land values, the

values currently assigned by the U.S. Forest Service will be employed. It is important to realize, however, that timber and watershed valuations are subject to argument, and total valuations may be too low because of the difficulty in assigning numbers to other factors, such as recreational activity and wildlife preservation.

Attempts to apply the theories to be developed here or in future work will be seriously handicapped unless better procedures are established for obtaining accurate and complete data from the field. One of the secondary purposes of the study is to prescribe the various types of data needed for least cost analyses and to make suggestions for their collection, for without good input data the value of the results obtained in actual practice will be diminished. The current lack of data does not impede the development of mathematical models; however, the application of such models for fire planning under actual field conditions will require extensive data now available only in fragmentary form. Most of the data required from future actual fire histories should be readily available, but procedures and responsibilities must be established for their collection.

The possible gains from study of least cost formulations are impressive. First, the results of such studies should provide a rational basis for planning which can be understood and acted upon by planners and budgetary authorities. It will be possible to effect cost tradeoffs between suppression and damage to give lower total costs than presently incurred. Also, realistic budgets could be set and adhered to with the need for "blank checks" associated with expenses for large campaign fires greatly reduced. In many protection areas large fires can no longer be tolerated, making adequate initial attack and control imperative. If this could be accomplished, the many problems and large expenses associated with large fires would be eliminated, or at

least vastly reduced. On the subject of suppression expenses, there is evidence [9] to suggest that increased manning for fire control will actually cause a reduction in total suppression expenditures, as well as in damage costs, again because of reducing the number of large fires and the huge suppression costs they cause. Such a consideration only increases the need for organized and detailed study of the problem of planning for initial attack.

D. General Relationship of Fire Size, Time to Control and Size of Suppression Force

It is generally agreed that both the final size of a fire and the elapsed time necessary to conrol it are inversely proportional to the size of the fighting force. Without attempting at this point to quantify these relationships, we might postulate curves as shown in Fig. 1.1. These curves will probably never reach either axis, as no matter how large a force is engaged in fire fighting, it will burn for some time and reach some size; and, at least within reasonable limits, if no, or very small, suppression activity is undertaken, a fire of the sort we are considering will burn indefinitely and increase in size without limits. Figure 1.1 is presented now, not in an attempt to anticipate future results, but to indicate a reasonable picture of suppression activity as obtained from actual field experience; any formulation which led to contradictions of this figure would certainly be suspect.

E. Firefighting Tactics

There are three basic tactics in current use for fighting wildland fires. These may be called the direct, indirect, and parallel tactics. In a particular fire situation, the choice of tactic depends on such factors as slope, terrain, fuel, and weather conditions. The direct tactic, used generally on slow-moving fires in relatively level terrain, consists of attacking

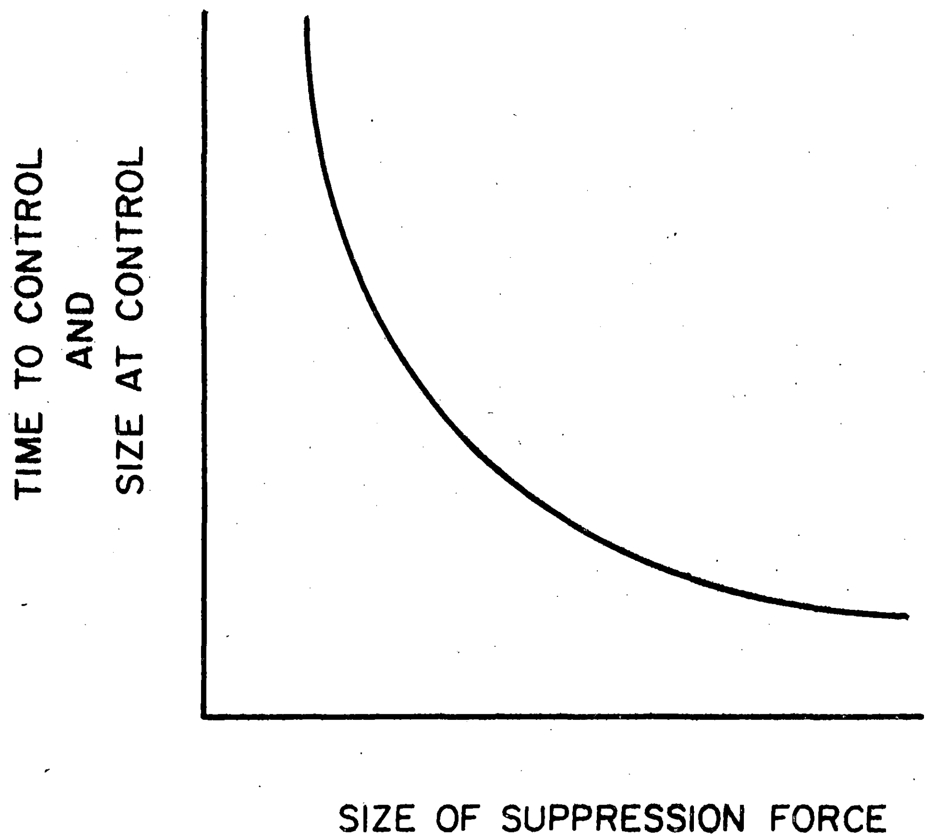

Figure 1.1 — General relationship of size of fire and time to control to size of suppression force.

the actual fire front directly and attempting to extinguish it before it spreads over any additional area. The indirect tactic, for use on fast-moving or crown fires in rough terrain or when natural breaks in the fuel exist, is to move back some distance away from the actual fire and construct a firebreak of some width in the fuel in hopes that the fire will burn itself out from lack of fuel when it reaches the break. The third available tactic, the parallel method, generally for use on fires of moderate intensity or in moderately steep terrain, consists of constructing firebreak from a point or points near the edge of the fire and continuing along the edges pinching off the front of the fire until finally it is completely contained by the break. In this paper we will develop and apply a direct attack model. Models for the other two tactics, as well as certain extensions to this direct attack model, can be found in [3].

F. Philosophy of Fire Growth Models

The mathematical model to be presented here cannot pretend to portray exactly the course of every wildland fire, nor to predict precisely the effect of various allocations and tactics of suppression forces. The immediate real benefit comes from taking the first steps toward the translation of a complicated physical phenomenon into quantitative terms.

This model will make use of certain parameters, such as growth rate, acceleration factor, and effectiveness, which are admittedly complicated functions of a great many other parameters. In fact, the solution to the problem of wildland fire control may be described as requiring the transformation of a large number of <u>physical</u> variables (i.e., wind, fuel, moisture content, etc.) into <u>operational</u> variables, such as the number of men to send, the type of equipment to use, and the speed of attack.

Since the model to be developed employs only a small number of physical variables, the transformation to operational decisions can be made

in a more organized and logical manner. The problem of transforming the large number of physical variables actually encountered in the field into the small number of physical variables used by this model will be only briefly considered here (see [4]), as this problem can be more reasonably attacked by those trained in combustion physics, fire weather, and related sciences. This research can be done unhampered by the necessity to consider operational factors; and, in fact, this division of the total problem should make both parts considerably more tractable.

Naturally, the model should be accepted only if the assumptions presented herein are substantially correct. Even if not all the features of a given fire are included, it should not be concluded that the results obtained are worthless; rather, the model will provide a first approximation to reality, to which later refinements will result in comparatively small changes. It attempts to describe in a "broad-brush" manner the general pattern of a fire, and a probable mechanism for its suppression. The results are rather general indications as to the economical sizes of suppression forces one should use for fires of various sizes and growth potentials, and, perhaps even more importantly, the effects of the various physical and economic parameters and groups of parameters upon the final costs of suppression and burn. This model also provide a framework in which various technological problems can be tentatively solved, and various new proposals ranked for further consideration. Further investigation and testing of the hypotheses of this model should provide a realistic basis for its ultimate use in decision making.

2. A DIRECT ATTACK MODEL

A. Introduction

This model, which for convenience will be called Model I, was first outlined in a paper by the author and W.S. Jewell [5] as a preliminary model for direct attack on wildland fires. Because of its continuity assumptions, it permits analysis by calculus optimizing techniques and can be used to describe the entire life of a fire from ignition to final extinguishment. Also, numerous extensions can readily be made to this model to describe more complicated fire situations (see [3]). However, this generality can also be considered a disadvantage for two reasons. First, some of the parameters described therein are difficult to evaluate experimentally, and second, the model does not readily allow consideration of specific tactical alternatives without further assumptions about fire geometry.

B. Formulation

The general pattern of fire growth can be pictured as in Fig. 2.1. At time T_I ignition of the fire takes place, but it is not detected until some time later, at T_D. Because of the delay in mobilizing and transporting men and equipment, the actual attack on the fire does not usually begin until an even later time, T_A. The size of the fire, which has been growing steadily or even accelerating up to this time, still continues to grow after the initial attack; but its rate of growth is slowly decreased until, at time T_C, the fire is brought under control with a total area of burn y_C. From T_C to T_M, mop-up of the burned area takes place, until final extinction occurs at T_F.

Thus, the ultimate area (and cost) of a fire is seen to depend upon

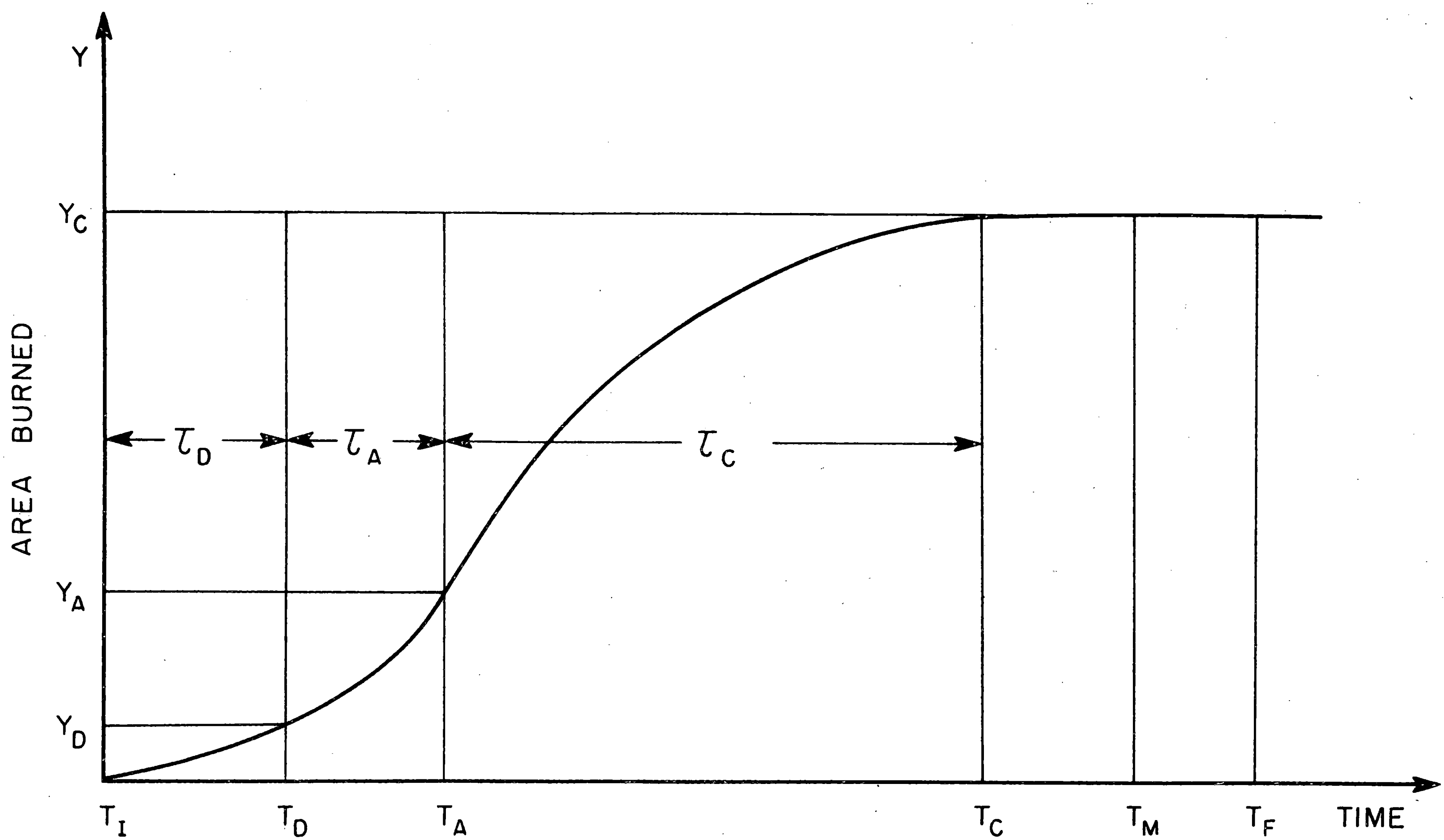

Figure 2.1 – Model I Fire growth

three critical time intervals. By spending more funds upon lookouts, aerial surveys, and other surveillance devices, the detection interval, τ_D, could be reduced. By investing in more rapid means of transport, such as helicopters, and by the staffing of seasonal or year-round ready crews, the attack interval, τ_A, may be shortened. Finally, by sending more suppression forces and equipment, the control interval, τ_C may usually be decreased. Mop-up usually proceeds at a leisurely pace, and thoroughness, rather than time elapsed, is a more important criterion for this operation.

At first we shall concentrate our attentions upon the time interval from T_A to T_C, the control interval, and will investigate the effect of allocating varying amounts of suppression forces on the fire. Later we will want to examine the attack interval to investigate the changes in total costs as this interval is changed. Since to some extent the length of this interval is under our control, we may be able to effect significant dollar savings by appropriate shortening of this time span. If the decision is to be based solely upon the parameters of the fire reported at the time of detection, T_D, then two physical "laws" of behavior are needed:

(1) A description of free-burning fire behavior during the interval from detection to attack.

(2) A description of the effect of a suppression force of a given size on a fire of known physical characteristics.

Since fire control theory is lacking in results in this area, we are forced to assume a "reasonable" representation of these "laws" and then compare the fire behavior thus predicted with intuition and experience. Hopefully, as various parameters are changed, or various limiting situations tested, this model will produce results in keeping with present fire control knowledge.

This model assumes that, in the absence of suppression forces, the growth of a fire consists of two deterministic components: a linear growth with time, and an accelerating component. Thus, if $t = 0$ represents time measured from time of detection, the growth of fire may be represented by:

$$\frac{dy}{dt} = G_D + Ht \tag{2.1}$$

$$y(0) = y_D \quad . \tag{2.2}$$

Here y_D is the observed area of burn at time T_D, G_D is the observed rate of growth (acres/hour, etc.), and H is acceleration component of growth (acres/hour^2). By an appropriate choice of parameters, one can choose a fire which is increasing its area in a linear fashion, or "simulate" one which is doubling in size every 24 hours, etc.

Since $T_A > T_D$ and dy/dt is an increasing function of t, at the time of attack, T_A, the fire has grown to a size

$$y_A = y(\tau_A) = y_D + G_D \tau_A + \tfrac{1}{2}H(\tau_A)^2 \tag{2.3}$$

and is growing at a rate

$$G_A = y'(\tau_A) = G_D + H\tau_A \quad . \tag{2.4}$$

which is faster than that observed at T_D due to the acceleration of the fire.

At the time T_A we have the possibility of sending in suppression forces. The effect of such forces in this model will be described as growth-rate decreasing, or "deaccelerating" forces. Thus, from the high point of growth rate at T_A, the forces slowly remove the momentum of the fire, until its growth rate is zero; at this point, the fire is under control. In mathematical terms, the growth rate after time T_A is described by:

$$(2.5) \qquad \frac{dy}{dt} = G_D + Ht - E(t - \tau_A)x \qquad t \geq \tau_A$$

$$= G_A - (Ex - H)(t - \tau_A) \quad .$$

That is, after T_A the fire has an effective deceleration $Ex - H$. Here x is a decision variable which represents the number of suppression forces sent in against the fire, and E is an "efficiency" factor which represents the effect of different types of forces, training, equipment available, etc. measured in area/man-time^2. This factor E will undoubtedly be a function of y_A, the fire size at time of initial attack, and may also be a function of other physical characteristics of the fire. It will depend on the number of chains (length), if not of the fire perimeter, at least of the fire front. In what follows we will use E as a constant, and refer only to fires that are "small" at time T_A, say, less than one acre.

We may think of this deceleration force in the following way: If the fire perimeter is moving at a linear rate, and a man is capable of extinguishing by various means a certain number of linear feet of fire per unit time, then when this force is greater than the growth spread, we have "effective deceleration."

From (2.5), we can solve directly for τ_C, the interval of time until control. We have:

$$(2.6) \qquad \tau_C = \frac{G_A}{(Ex - H)} = \frac{(G_D + H\tau_A)}{(Ex - H)} \quad .$$

By integrating (2.6), we can find the additional area burned during the control interval, letting $u = t - \tau_A$:

$$y_C - y_A = \int_{\tau_A}^{\tau_C+\tau_A} \frac{dy}{dt}\,dt = \int_0^{\tau_C} \left\{G_A - (Ex - H)u\right\} du = G_A\tau_C - \frac{(Ex - H)(\tau_C)}{2}$$

and substituting (2.6), we have:

$$(2.7) \qquad y_C - y_A = \frac{(G_A)^2}{2(Ex - H)} .$$

It is seen that the growth is composed of two parabolic portions, one where the fire spreads and accelerates, burning freely during the attack interval, and a decreasing parabola where the fire is being slowed down by the suppression forces.

Notice that the fire cannot be brought under control at all unless Ex is greater than H. Thus, there is some minimal suppression force

$$(2.8) \qquad x_0 = \frac{H}{E}$$

which is required. From (2.5), it can be seen that this minimal force is just the deceleration factor which "kills off" the acceleration of the fire, and straightens out the growth curve into a straight line; we might say that these men are keeping the growth rate "under control," although they are certainly not preventing the fire from spreading — additional manpower is required to do that.

If we use the variable z to denote the number of forces above and beyond x_0, i.e.,

$$(2.9) \qquad z = x - x_0 ,$$

then we see that the control interval becomes:

$$(2.10) \qquad \tau_C = \frac{G_A}{Ez}$$

$$(2.11) \qquad y_C - y_A = \frac{(G_A)^2}{2Ez} \quad .$$

Both of these quantities are inversely proportional to the number of additional men employed; thus both curves have the form shown in Fig. 1.1. This result states that there is some minimal number of forces which is needed to make any progress against the fire at all; beyond that point there is an inverse relationship between the number of forces and the time and area of burn.

In actual practice, forest personnel may prefer to estimate the approximate shape of a curve like Fig. 1.1, rather than estimate the individual parameters G_A, H, E, etc. Our purpose has been to show that a simple operational model could lead to a realistic description of system behavior.

The influence of the effectiveness of parameter E in (2.6) and (2.7) is not surprising, since E multiplies the manpower to get fire deceleration. However, the growth rate of fire at attack time, G_A, enters in two different ways. Equation (2.6) states that the control interval is directly proportional to G_A; in (2.7), however, we find that the ultimate area of burn will be proportional to the square of the growth rate at attack (for the same number of men)! States in other words, if we have two fires, one of which is spreading twice as fast as the other, then we need twice as many men (above and beyond x_0) to put out the fire in the same time; but if our object is to confine the fire to the same area, then we need four times as many additional men. G_A also contains the effect of attack time through the relationship

$G_A = G_D + H\tau_A$. We shall investigate this dependence later.

C. Economics

Making the correspondence with (1.1), we have

(2.12) $$C = C_F + C_S x + C_T \tau_C + C_B y_C + C_X \tau_C x$$

or

(2.13) $$C = C_F + C_S x + C_X \frac{G_A}{Ex - H} x + C_B \left[y_A + \frac{G_A^2}{2(Ex - H)} \right] + C_T \left[\frac{G_A}{Ex - H} \right]$$

where y_A is given by (2.3).

We note that C_S will depend, to a large extent, on the method of transport used (foot vs. truck vs. helicopter, etc.), and this in turn will influence the interval between detection and attack, τ_A. Later we will want to examine the effect on the total costs of varying C_S, and hence τ_A; for the present we merely point out that there is a functional relationship between the two quantities. We further note that C_T may be removed from (2.13) by combining $C_T[G_A/(Ex - H)]$ with $C_B[G_A^2/2(Ex - H)]$ to obtain a new C_B, say C_B' which equals $C_B + (2C_T/G_A)$.

Using the relationship (2.9) to eliminate the fixed base of suppression force which must be sent anyway, we find that the total cost as a function of the additional forces sent may be written as:

(2.14) $$C(z) = C_0 + C_S z + \frac{(C_X G_A H/E^2) + (C_B' G_A^2/2E)}{z}$$

where

$$(2.15) \qquad C_0 = C_F + \frac{C_S H}{E} + \frac{C_X G_A}{E} + C_B y_A$$

is a constant cost, independent of the number of additional forces sent.

We may readily identify the various portions of (2.14) in their effect upon the total cost if various actions are taken. For instance, if one sends in a large number of suppression forces z will be large and the second term in (2.14) will be dominant; that is, our cost will be due largely to travel and other one shot items which are proportional to the size of the force which is employed. On the other hand, if few men are sent in, then the last term in (2.14) becomes dominant, and two factors are contributing to our cost — the cost of the burned area, and the manpower-hour cost. It is interesting to note that manpower-hour costs are higher the fewer additional men are sent; this is because the time that the total forces must spend extinguishing the fire is determined disproportionately by the few extra men, thus increasing total manpower-hours.

D. Optimization

To find the otpimal level of suppression, we take a short mathematical detour to obtain some results in the mathematical theory of optimization which apply to the special form of (2.14).

THEOREM: If $C(z) = \alpha z + \beta/z + \gamma$, then $C^* = C(z^*) = \underset{z=z^*}{\text{minimum}}\, C(z)$ is attained at the point $z^* = \sqrt{\beta/\alpha}$ with $C^* = 2\sqrt{\alpha\beta} + \gamma$. Furthermore, $C''(z^*) = 2\alpha\sqrt{\alpha/\beta}$, and $C'''(z^*) = -6\alpha^2/\beta$.

PROOF: The necessary condition for a stationary point is that dC/dz be zero, i.e., $\alpha - \beta/z^2 = 0$. This occurs at the points $z = \pm\sqrt{\beta/\alpha}$. The

second derivative is $d^2C/dz^2 = 2\beta/z^3$, and a sufficient condition is that the stationary point just found is a minimum in that $C''(z^*) > 0$. This occurs at $Z^* = +\sqrt{\beta/\alpha}$, with $C''(z^*) = 2\alpha\sqrt{\alpha/\beta}$, and $C'''(z^*) = -6/z^4 = -6\alpha^2/\beta$. Upon substitution, we find that the two variable components of the cost function are equal at optimality, with $C^* = 2\sqrt{\alpha\beta} + \gamma$.

With these results, we can find the optimal initial attack force from (2.14) after making a correspondence between the various constants.

We find that the optimal number of additional forces to be sent to the fire is:

$$(2.16) \qquad z^* = \sqrt{\frac{(C_X G_A H/E^2) + (C_B' G_A^2/2E)}{C_S}} = \sqrt{\frac{C_B' G_A^2 E + 2C_X G_A H}{2E^2 C_S}} \; .$$

In most operations the cost rate of manpower-hours is small compared to the rate at which wildland values are burned. By defining ε_X as the ratio:

$$(2.17) \qquad \varepsilon_X = \frac{C_X H}{C_B' G_A E} = \frac{C_X x_0}{C_B' G_A}$$

(i.e., the ratio of cost rate for the basic minimal force to the cost rate of burning wildland at time of attack), we can rearrange (2.16) as:

$$(2.18) \qquad z^* = G_A \sqrt{\frac{C_B'}{2C_S E}} \sqrt{1 + 2\varepsilon_X}$$

where, if $\varepsilon_X << 1.0$, $\sqrt{1 + 2\;_X} \approx 1 + \varepsilon_X$, a small correction term. The optimal total suppression force is, of course,

$$(2.19) \qquad x^* = z^* + x_0 = z^* + (H/E) \; .$$

In fact, in much of the theoretical and numerical work to follow we shall take ε_X as equal to zero, a procedure justified by the realization that in almost all practical situations C'_B will be much greater than C_X and for most fires x_0 will be small.

By substitution, we find that the minimal cost incurred by sending the optimal number of suppression forces is:

$$C^* = C_0 + 2G_A \sqrt{\frac{C_S C'_B}{2E}} \sqrt{1 + 2\varepsilon_X} \quad . \tag{2.20}$$

Later, we will examine the effect of sending a nonoptimal number of men on an initial attack, both too few and too many. For this purpose, we also record at this point the second and third derivatives of $C(z)$ at the optimal point $z = z^*$.

$$C''(z^*) = \frac{2}{G_A} \sqrt{\frac{2EC_S^2}{C'_B}} \frac{1}{\sqrt{1 + 2\varepsilon_X}} \tag{2.21}$$

$$C'''(z^*) = -\frac{12\,EC_S^2}{C'_B G_A^2} \frac{1}{(1 + 2\varepsilon_X)} \tag{2.22}$$

A rather interesting result occurs in the solution for the control interval τ_C, if an optimal policy is followed. We have:

$$\tau_C^* = \frac{G_A}{Ez^*} = \sqrt{\frac{2C_S}{C'_B E}} \frac{1}{\sqrt{1 + 2\varepsilon_X}} \quad . \tag{2.23}$$

Except for possible variations in ε_X and a possible functional relationship between E and G_A, τ_C^* does not depend on G_A directly; that is, for given equal values of C_S, C'_B, and E, the optimal policy keeps (almost) all

fires burning for the same length of time after the attack of suppression forces.

NUMERICAL EXAMPLE

If a fire, when detected, were observed to have the parameters

$$G_D = 3.5 \text{ acres/hour} \quad \text{and} \quad H = 3.0 \text{ acres/hour}^2$$

then in the next half hour, the fire would consume an additional 2 1/8 acres, and would then be burning at a rate of spread 5.0 acres/hour. In the next half hour after that the fire, if unchecked, will burn 2 7/8 more acres, and be growing at a rate of 6.5 acres/hour. Let $E = 0.1$ acres/hours^2-man, $\tau_A = 0.5$ hours, and consider sending a suppression force of 44 , 54 , or 64 men. By direct calculation, we see that the control interval, and (controllable) area of burn are:

	44 men	54 men	64 men	
Control Interval τ_C	3.57	2.08	1.77	hours
(Controllable) Area of Burn $y_C - y_A$	8.93	5.21	3.68	acres

Thus, by putting more suppression forces into the fire, both the time to control and the total acreage burned are decreased; this is certainly in agreement with actual experience and intuition. Compare with Fig. 1.1.

Now further assume that $y_D = 1.0$ acre and that the various cost parameters for this particular fire are:

$C_F = 1{,}000$ dollars $\qquad C_S = 50$ dollars/man

$C_X = 3$ dollars/man-hour $\qquad C_B = 200$ dollars/acre $= C_B'$ (including mop-up) i.e., $C_T = 0$

We may take $C_T = 0$ for two reasons. First, consultation with experienced fire control personnel has failed to discover any fire cost items with these dimensions that could be included in this term; and secondly, we will see in Chapter VI that because τ_C^* is invariably quite small, C_T would have to be quite large before $C_T \tau_C^*$ could have a significant effect on total costs.

From the previous calculations, we see that at the time af attack, a total of $y_A = 1 + 2\ 1/8 = 3\ 1/8$ acres will be burning, with an instantaneous growth rate of $G_A = 3.5 + 3/2 = 5.0$ acres/hour. The required minimal

number of forces is $x_0 = 3/0.1 = 30$ men. From (2.14), we calculate that the fixed cost of operation will be:

$$C_0 = 1,000 + (50 \times 30) + \left(\frac{3 \times 5}{0.1}\right) + \left(\frac{200 \times 25}{8}\right) = \$3,275 \quad .$$

Also,

$$C'_B G_A^2/2E = 25,000 \text{ dollar-men}$$
$$C_X G_A H/E^2 = 4,500 \text{ dollar-men}$$
$$\varepsilon_X = 0.09 \quad .$$

The total cost of fire suppression is:

$$C(z) = 3,275 + 50z + 29,500/z \text{ dollars} \quad .$$

The cost function is plotted in Fig. 2.2 versus z, and the individual components are also shown and labelled as individual curves. From (2.18) we calculate:

$$z^* = 5\sqrt{20}\ \sqrt{1.18} = 24.3 \text{ men}$$

are required above and beyond x_0, making a total of about $x^* = 54$ men to be sent to the fire.

When this is done, \$1,215 additional dollars are spent for transportation and other one-time expenses, about \$1,030 dollars are lost in burned acreage past the time of attack, and about \$185 dollars are spent in manpower-hour dollars, making a total (minimal) variable cost of \$2,430 dollars, for a grand total minimum cost of $C^* = \$5,705$ dollars. After breaking down the fixed cost component C_0, we find that the total cost was spent as follows:

Fixed Cost of Operation	\$1,000
Transportation-type Expenses	2,715
Manpower-hour Expenses	335
Total Burned Acreage Cost	1,655
	\$5,705

The time to control the fire is about $\tau_C = 2.06$ hours, with a total area of burn of about $y_C = 8.28$ acres. The progress of the fire is shown as the middle curve in Fig. 2.3. (The slight discrepancies with the table above are due to the fact that exactly 54 men were used in the calculation.)

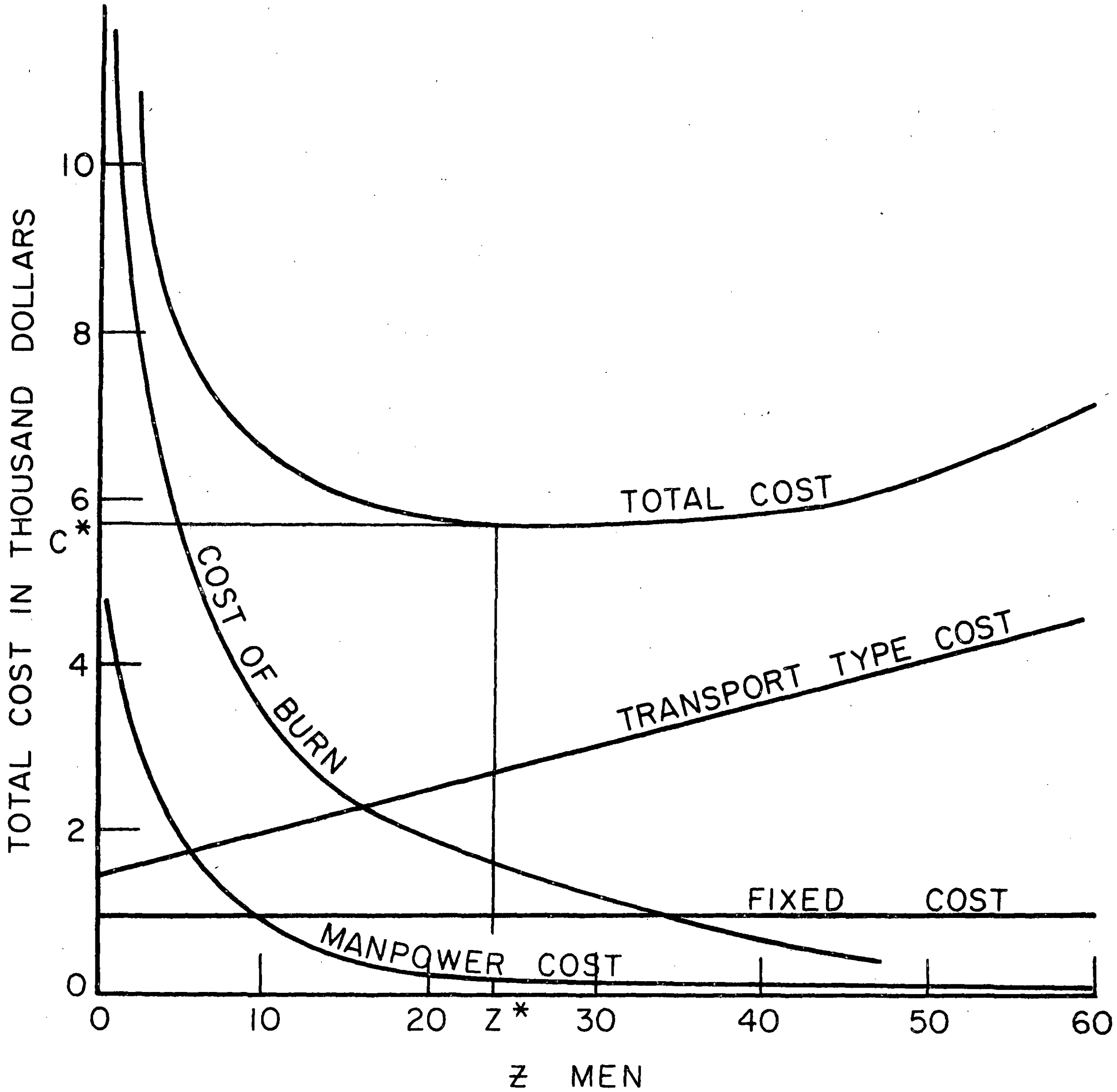

Figure 2.2 — Total cost and its components

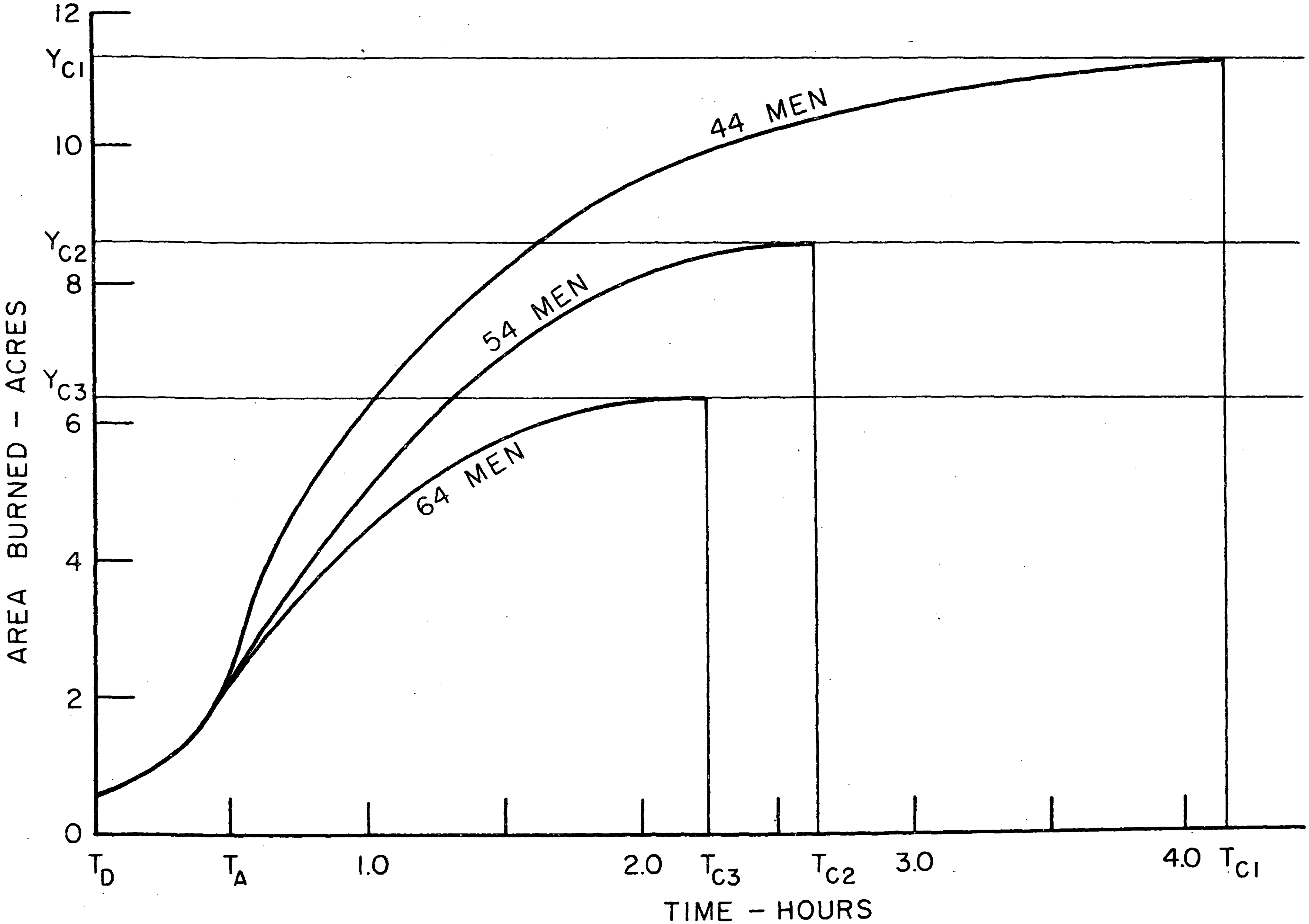

Figure 2.3 — Fire patterns for different numbers of men.

This example is intended as a representative one only, and should not be interpreted as "realistic," or "unrealistic."

E. Extensions of the Model

This subject is covered more fully in [3]. Here we merely mention two of the results that can be obtained.

First, in the absence of suppression forces, we have assumed that fire growth can be described by a function $f(t) = G + Ht$. We have then developed various relationships between costs and size of suppression forces by further assuming these forces act as an effective decelerating force, so that fire growth is described by $G + Ht - Ext$. We have been led to various conclusions, one of the most important being that over and above the minimal suppression force x_0 that must be sent anyway, the optimal strategy occurs when the transportation type expenses are just balanced by the sum of the burned area costs plus the manpower-hour costs. As mentioned above, this result was obtained by taking $f(t) = G + Ht$. If we remove this restriction and assume only a monotone increasing function $f(t)$, (such that the first derivative is always positive), to describe fire growth in the absence of suppression forces, (thus, we may allow H to be negative for limited periods of time) we find

(2.24) $$\tau_C^* = \sqrt{\frac{2C_S}{C_B E}}$$ to be independent of $f(t)$.

This result has already been obtained in (2.23) for the special case $f(t) = G + Ht$, but now we see it to be true for a more general $f(t)$.

Also, we find that the savings due to the men are equal to their costs, and the result obtained earlier of balancing suppression costs and damage costs is seen to hold no matter what the form of $f(t)$, as long as it meets the above restrictions.

Secondly, consider a variation of Model I where we work with perimeter increase rather than area increase. Although the algebra is more formidable, the only new theoretical difficulty encountered is the necessity of making some assumption about fire geometry. For their statistical purposes, the U.S. Forest Service assumes that final fire area can be approximated by an ellipse [10], most commonly with one axis twice the other. We will assume that fire grows as an ellipse with one axis n times the other, where n can be any positive real number. Taking n equal to two would give the most common fire configuration [10].

Following the same general procedure as with the area increase model, we readily obtain a cubic equation in x (as a function of n), which may be solved for x^*, and thus C^* can be obtained by substitution.

We turn now to the application of the basic Model I to actual fire situations.

3. ANALYSIS OF ACTUAL FIRE HISTORY

A. Introduction and Choice of Area

The numerical example in the preceding section suggests that the optimum number of men to send to a given fire may be considerably larger than current practices indicate. This implies that the number of fire-fighters available for initial attack should perhaps be considerably increased in order to incur lower total costs. In this section the actual fire history for one year on one National Forest will be compared with the optimal policies given by Model I for the same fires to examine just how many men would be required and how the actual total costs incurred compare with the theoretical total costs as given by the model.

The fire history for the calendar year 1959 on the Plumas National Forest will be used for this analysis. The Plumas was chosen because of its relatively uniform resource valuations, the keen interest of its operating personnel in fire control research, and the availability of published results of previous research efforts on this forest.[2], [9], [1] The year 1959 was selected because of its fairly active fire history and its proximity to the base period of 1950-1958 from which actual data will be used [4]. The burned acreage during 1959 on the Plumas National Forest was approximately 11,500 acres, while for the decade 1950-1959 the burned acreage ranged from a low of 97 in 1958 to a high of 24,128 in 1951, with a yearly average of 6,325. The total number of fires in 1959 was 139, while this figure ranged from 62 in 1954 to 333 in 1951 with an average of 165 for the same decade.

B. Actual Fire History

These data were obtained from the original 929 forms[13] on file with the U.S. Forest Service. The items required for this analysis were punched on IBM cards, the special programs written, and computation done on an IBM 1620 computer. Estimates of the computation time required will be given where appropriate. The actual fire history for 1959 on the Plumas National Forest is shown in Table 3.1.

Table 3.1

SIZE in acres	CLASS	FIRE HISTORY		
		Number of man-caused fires	Number of lightning-caused fires	Total
< 1/4	A	36	68	104
1/4 - 10	B	14	9	23
10 - 100	C	6	1	7
100 - 300	D	1	0	1
> 300	E	4	0	4
		61	78	139

Assuming that each Class A fire burned an area of one-eighth (1/8) acre, these fires consumed a total of 11,477.75 acres. To determine the dollar damages caused by these fires two maps were prepared, the first showing the location and general outline of all 139 fires, and the second showing the resource valuations for the entire forest area. This second map was prepared using the guides outlined in the Forest Service Handbook[8] for evaluating various forest resources. There are six possible value classes into which a forest area may fall: Class 1, with a value of from 0-50 dollars/acre; Class 2, 51-200; Class 3, 201-500; Class 4, 501-1,000;

Class 5, 1,001-2,000; and Class 6, over 2,000 dollars per acre. To attach a specific numerical value to each area, the midpoint of each class was taken, except for Class 6 where the value of 2,000 dollars/acre was used. From these two maps, the total dollar damages caused by fire during 1959 on the Plumas National Forest was determined as shown in Table 3.2.

Table 3.2

Resource Value Class	$ Value/Acre	Acres Burned	Total $ Damages
1	25	45.25	1,131.00
2	125	14.88	1,859.00
3	350	11,277.13	3,946,994.00
4	750	12.37	9,281.00
5	1,500	10.50	15,750.00
6	2,000	117.62	235,250.00
		11,477.75	4,210,265.00

To this sum the costs of suppression must be added to arrive at the total costs against which the total predicted by the model will be compared. Suppression costs only will be considered, so this figure will not be the total cost of fire; pre-suppression, prevention, and certain overhead expenditures will not be included either in the actual or theoretical totals. Examination and tabulation of cost records available at the Forest Headquarters, Quincy, California, indicates a total suppression cost of $754,394.00, so for these purposes the total costs, to the nearest thousand dollars, are $4,965,000.00. This does <u>not</u> include firefighter standby time, as manpower costs are only charged to suppression expenditures when the men actually engage in firefighting.

C. Assumptions for Use of Model I

To apply Model I to these fires, it was necessary to obtain values for the various parameters. Each fire was first classified according to its cause—fuel—fire danger rating combination. If the fire size at time of attack was reported as anything greater than zero, new values of G and H (and E if no reinforcements were used), were calculated from the existing fire data by the procedures described in [4]; if the size at attack was zero, H was taken as zero, and the G and E values were taken from the tables in [4]. Values for E when reinforcements were used and size at attack was not zero were also taken from the appendix. If the fire was a Class A fire, Class A growth rates were used. If the fire danger rating was reported unknown, the value for the closest known day was taken; and if no G or E values were available from [4] for the actual fire danger rating, the values for the closest fire danger rating where values were available were employed. At this point it was necessary to eliminate three of the fires from further analysis — two of them because the model did not seem to fit the reported data at all and the other because there was no information available on the particular fuel type. Since the results of this analysis will indicate the necessity of increased manpower, it is assumed that these three fires could be controlled at least as quickly with the increased manning as they actually were. Hence, these three fires will be treated as theoretical equals actual. The other 136 fires, however, readily permit analysis by the Model I.

The fixed costs parameter C_F, will be taken as zero in the individual fire analyses, and are added later as functions of the overall level of activity of the suppression organization. C_T, the costs per unit time will also be taken as zero for the reasons indicated earlier. Values of C_B,

the burn costs, or damages, were taken from the aforementioned resource valuation map, after plotting the point of origin for each fire. For the first analysis, C_X, the manpower-hour cost, was taken as $4 per man-hour, as shown on Form 261-R5, the 10 A.M. Fire Report,[12] of the U.S. Forest Service. To find C_S, the transportation costs, it is assumed that all transportation is by pick-up truck, with an operating cost of $11\frac{1}{2}$ cents per mile. This figure was multiplied by the round trip mileage to each fire and the result divided by the capacity of the pick-up (3 men) to give the C_S value for each particular fire. It is assumed here that all initial attack is by men with hand tools, and that in each instance the attack could be made in the same time as actual travel time. The only difference is that the attacking forces are of different strength; i.e., attack is by a different number of men. In specific instances, initial attack might have been made by bulldozers (etc.) at an even lower total cost, but here it is assumed that only men and hand tools are available.

D. Unconstrained Optimum Solutions

1. Calculation Procedure

Each of the 136 fires was then analyzed by equations (2.16) and (2.20) to find x^*, the optimal number of men to send to each fire. Table 3.3 summarizes the results of these calculations, and Table 3.4 shows how the optimal values of x^* compare with the actual number of men sent on initial attack. Since we have seen it to be generally better to overkill than underkill a fire, all noninteger solutions for x^* have been rounded up to the nearest integer. These calculations required approximately thirty minutes computer time for input, computation and output printing for the 136 fires, or approximately 0.22 min/fire.

2. Results

Table 3.3

Number of Fires Categorized by Value of x^*

x^*	CAUSE		TOTAL
	Man	Lightning	
1-5	9	46	55
6-10	14	15	29
11-20	12	6	18
21-50	10	4	14
51-100	4	2	6
101-200	2	5	7
201-500	6	0	6
over 500	1	0	1
TOTAL	58	78	136

Table 3.4

Initial Attack Manning—Theoretical vs. Actual

CAUSE	CLASS	UNDER MANNED	OVER MANNED	EVEN	TOTAL
Man	A	29	3	3	35
	B	13	1	0	14
	C	5	0	0	5
	D	0	0	0	0
	E	4	0	0	4
Lightning	A	41	15	12	68
	B	9	0	0	9
	C	1	0	0	1
	D	0	0	0	0
	E	0	0	0	0
TOTAL		102	19	15	136

Note that not only were most fires undermanned (approximately 75%) but with only one exception all fires larger than Class A were undermanned. Figures 3.1 and 3.2 show scatter diagrams of the actual number of men sent on initial attack vs. the unconstrained x^*'s for Class A fires and Class B, C, D and E fires respectively.

It is interesting and important to note that with the increased manning, the problem of simultaneous fires does not occur, since the optimal control times are so short, except in two instances of lightning fires, where the sum of the x^*'s is still well below the highest overall x^*. If it is assumed that unlimited manpower is available, and only must be paid when actually fighting fires, an unconstrained solution C^* may be found from (2.20) for each fire, and then summed over all fires to get the total suppression costs. These results are summarized in Table 3.5.

Table 3.5

Unconstrained Costs and Area from Model

	Total final area (acres)	Total unconstrained damage costs ($)	Total unconstrained suppression costs ($)	Total unconstrained costs ($)
136 analyzed fires	35.19	21,932.00	5,090.00	27,022.00
3 omitted fires	182.13	49,270.00	22,450.00	71,720.00
Total	217.32	71,202.00	27,540.00	98,742.00

The total cost here is approximately $99,000, which when compared to the actual total of $4,965,000 gives a savings of $4,866,000 by increasing the available manpower to the optimal levels on each fire. This result is clearly not realistic, since it would not be possible to have this many men available for fire fighting without paying a considerable amount of standby

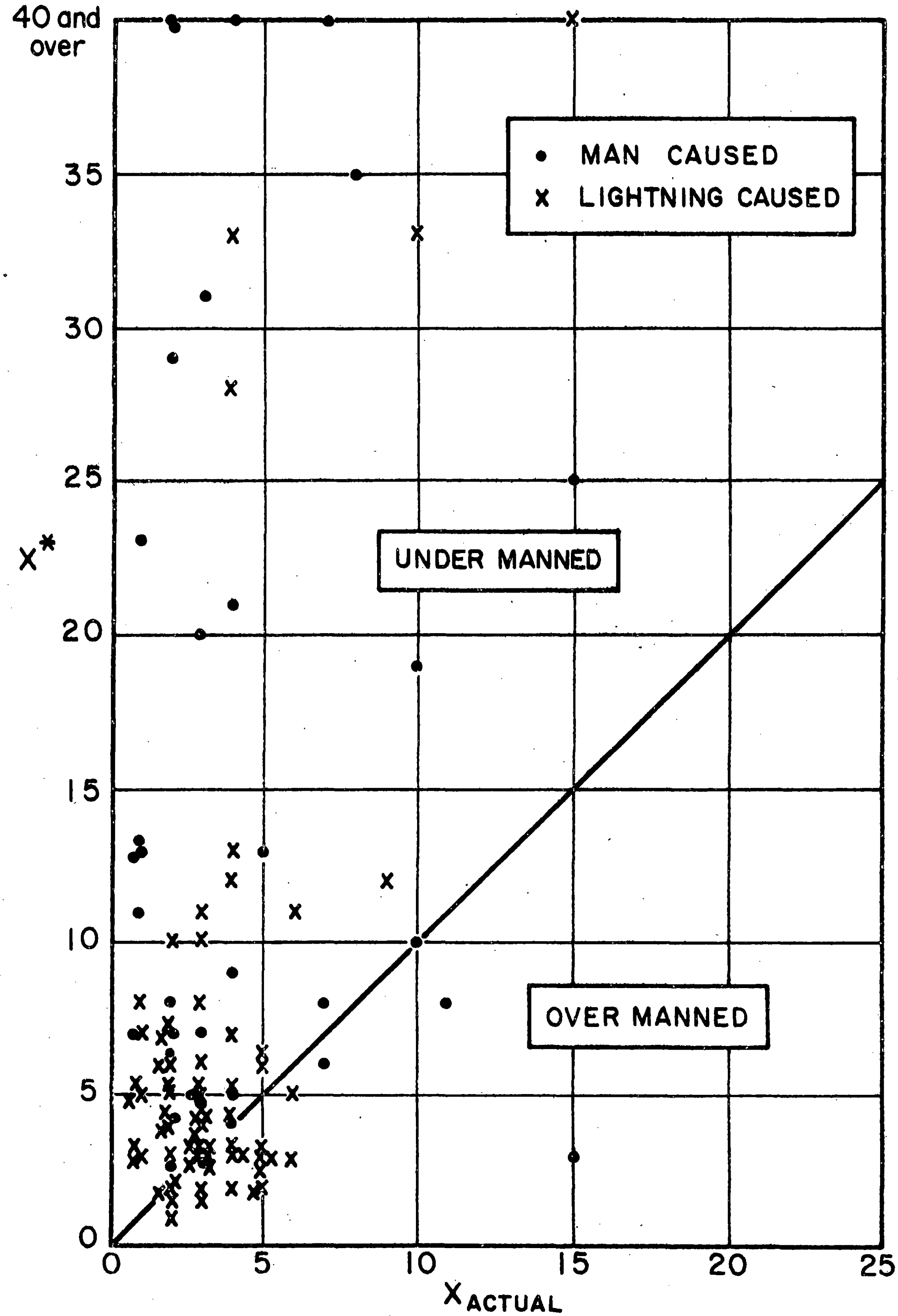

Figure 3.1 — Scatter diagram - Class A Fires.

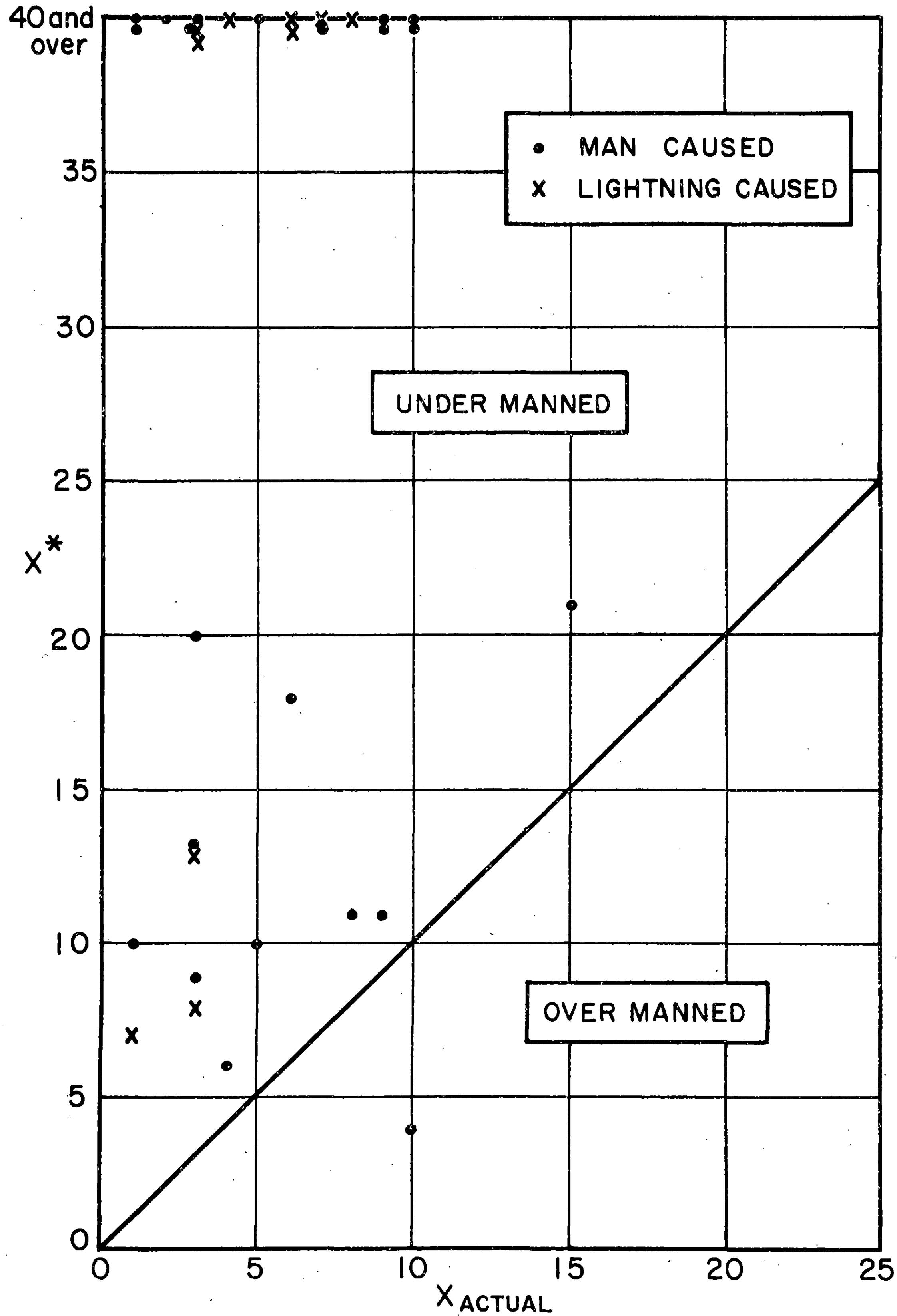

Figure 3.2 — Scatter diagram - Class BCD and E fires.

time. Clearly it will be cheaper in the long run to use a nonoptimal number of forces on some of the fires, letting them become larger than y^*, thereby reducing the total number of men available and hence the standby costs that must be incurred.

E. Constrained Optimum Solution

1. Further Assumptions

To pursue this point, and to make this comparison considerably more realistic, some assumptions will be made about the possible effect of manpower constraints.

a) The fire year is divided into three periods, April 1 to May 31, June 1 to October 10, and October 11 to December 14, the end of the fire season on the Plumas National Forest in 1959. We assume the number of permanent firefighters can be adjusted at the beginning of each period, but once a man starts a period his position is filled for the entire period. Firefighters are on standby 12 hours a day 7 days a week.

b) The standby cost of a firefighter is $2.02 per hour. This figure is arrived at by computing a weighted average of the pay rates for the various classes of firefighters and adding an allowance of 18 cents per hour to cover uniforms, maintenance of quarters, communications and radio, utilities, supplies and equipment, clerical workers, and leave. These figures were obtained from actual operating records of the Plumas National Forest. Thus, to fill one firefighter position for the April 1 – May 30 period incurs a fixed cost of 61 days times 12 hours per day times $2.02 per hour equals $1,478.64. Similar calculations apply to the other two periods.

c) When the man is actually engaged in firefighting, an additional

cost of $4 less $2.02 equals $1.98 per man hour is incurred, the $4 figure again coming from the standard charge on the 10 A.M. Fire Report.

d) Fire is equally likely on each of the six ranger districts making up the Plumas National Forest, so the same size initial attack force must be maintained on each ranger district. Therefore, the optimal number of men will be calculated as if all fires occurred on the same ranger district and then the actual number of men required will be found by multiplying this number by six. The firefighter fixed costs must also of course be multiplied by six.

With these assumptions we now solve a constrained problem, balancing manpower costs, including standby, and total costs. Thus some fires will be allowed to burn larger than in the unconstrained case, and these extra damage costs will be balanced by having a smaller suppression organization and hence lower standby costs. The objective is to find the optimal number of firefighter positions to be filled for each of the three time periods such that the sum of all manpower, transportation, and damage costs is minimum.

2. Calculation Procedure

For each time period:

1. Select some value of x, the number of firefighter positions to be filled.
2. For each fire during time period calculate the unconstrained x^*.
3. If $x^* \leq x$, round x^* to next highest integer and calculate total costs.
4. If $x^* \geq x$, calculate the (suboptimal) total cost of the fire using only x men.
5. Sum all cost totals from Steps 3 and 4.
6. Calculate fixed costs of men (1,478.64 times x for the first time period).
7. Add fixed costs from Step 6 to sum from Step 5 to obtain grand total sum of costs for the period.
8. Change x and repeat from Step 1 until a minimum has been obtained at Step 7.

Computation times for each iteration (value of x) were approximately two minutes for time periods one and three, and fifteen minutes for time period two, the main fire season. The difference is due to the fact that there were 118 fires during period two, and only 8 in period one and 10 in period three.

3. Results

Table 3.6 gives the results of these calculations, and Figs. 3.3, 3.4, and 3.5 show the effect of varying x from its overall optimal value for each of the three periods. Table 3.7 presents the breakdown into components of the total costs presented in these figures. Again it is seen that it is far better to have too many men than too few.

Table 3.6

Overall Constrained Optimum Costs

Period	Number of firefighter positions	Cost of men	Acreage burned	Burn Cost	Total Cost
Apr. 1-May 31	13	19,222.32	23.43	8,610.50	30,011.31
June 1-Oct. 10	25	79,992.00	63.03	28,944.82	110,668.23
Oct. 11-Dec. 14	23	36,238.80	12.81	17,079.21	53,792.09
TOTAL		135,453.12	99.27	54,634.53	194,471.63

As mentioned above, these costs are figured on the basis of one ranger district. Therefore, to get the true total costs, the manpower costs must be multiplied by five to cover the other ranger districts and this figure must be added to the sum of the total costs given in Table 3.6. The total thus obtained is (135,453.12 times 5) plus 194,471.63, or 871,737.23 dollars. To this must be added the actual costs of the three fires not included in the analysis, which, from Table 3.5 are $71,720.00.

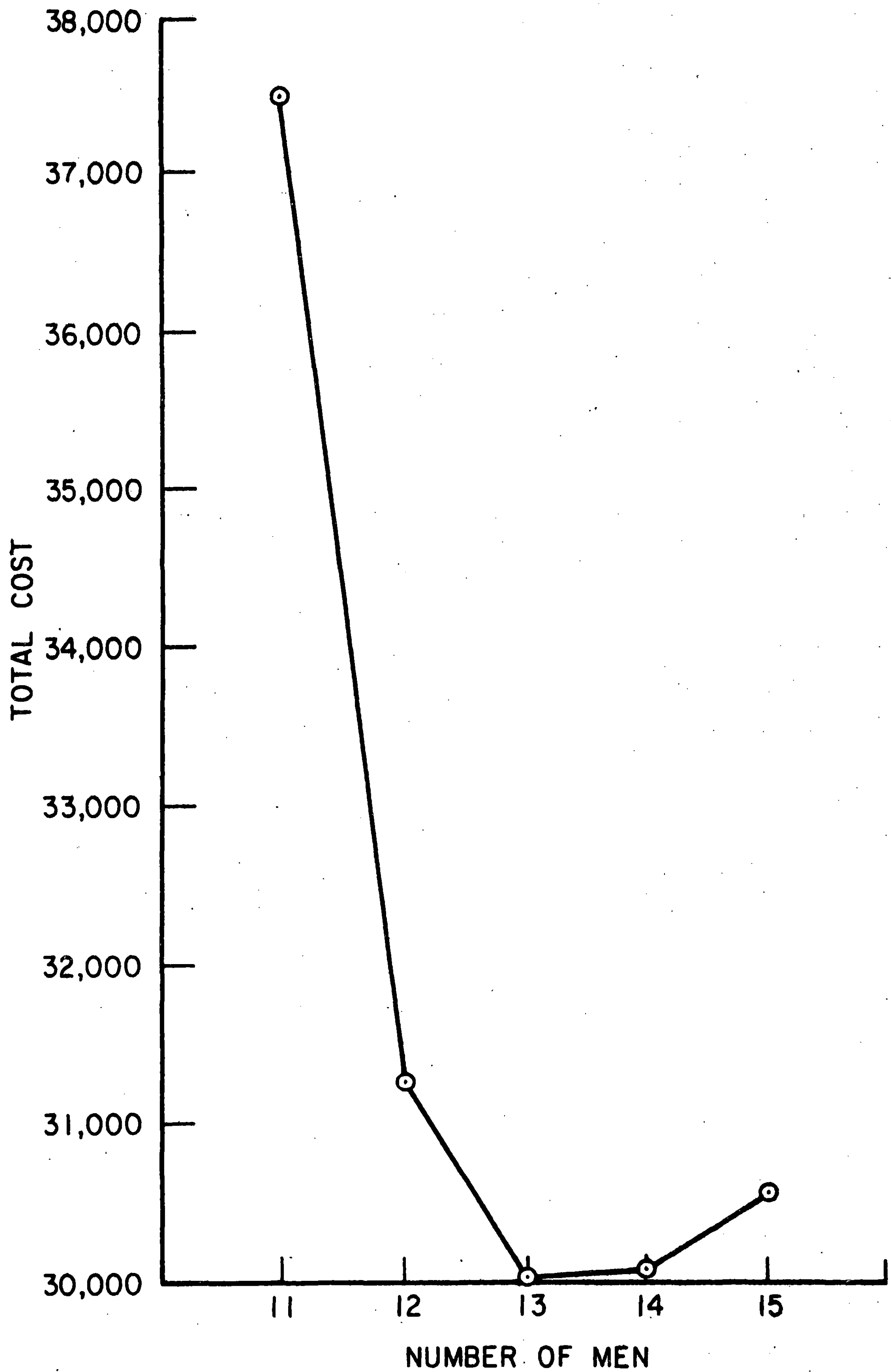

Figure 3.3 — Total cost from Model - April 1 - May 31.

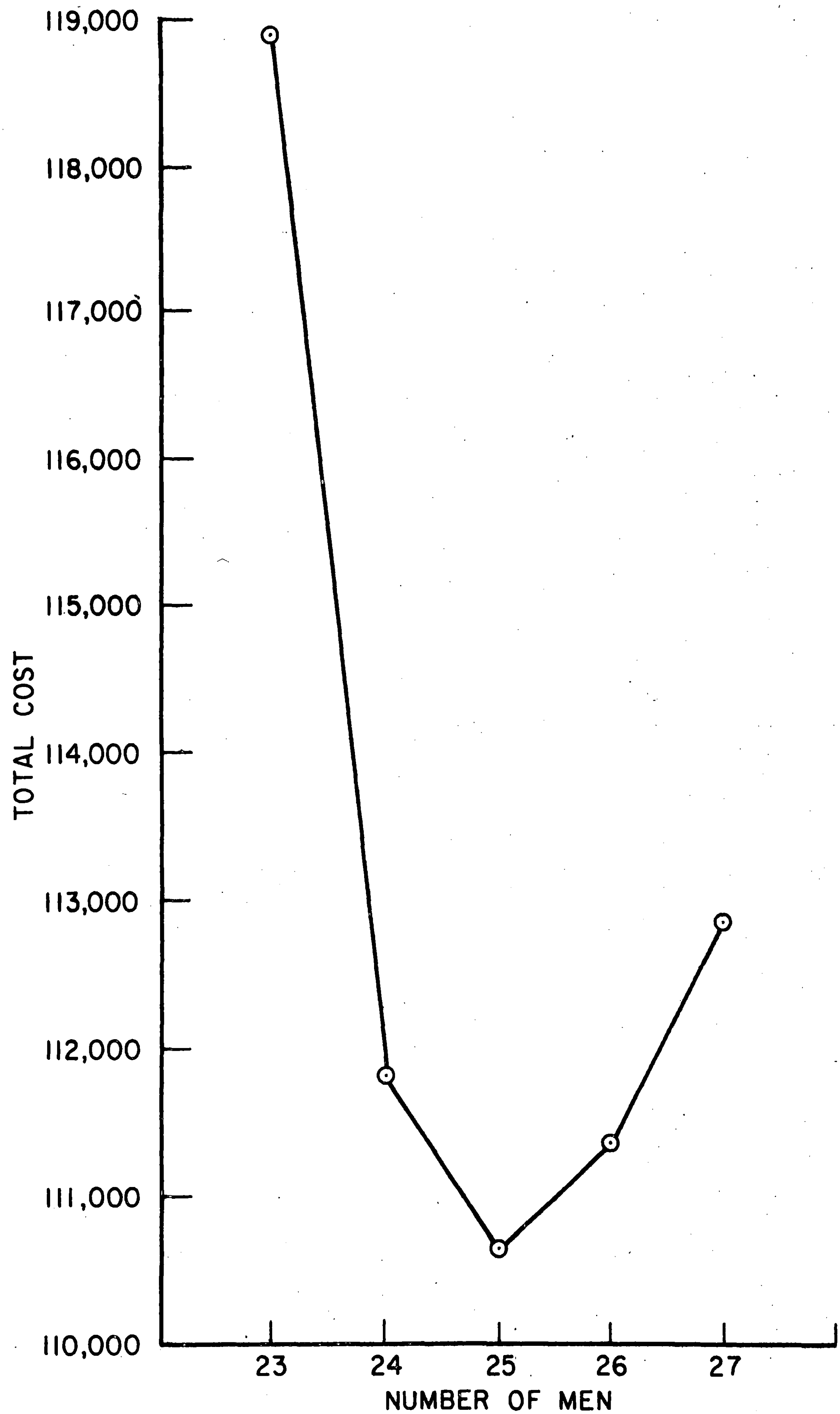

Figure 3.4 -- Total costs from Model - June 1 - October 10

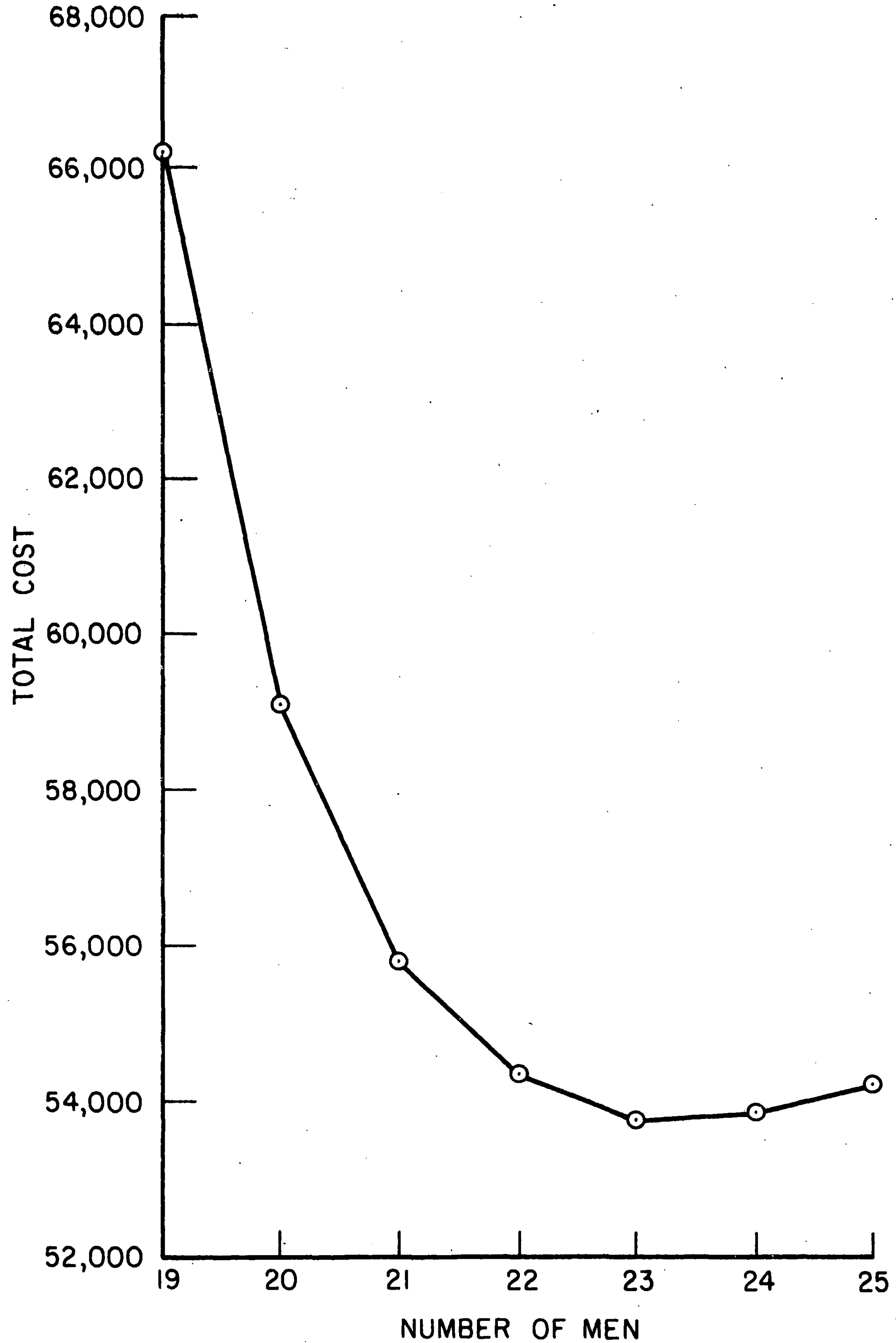

Figure 3.5 — Total costs from Model - October 11 - December 14

Table 3.7

Total Cost and Its Components (in dollars)

Period	No. of men	Burn Costs	Costs of men	Suppression Costs	Total
Apr. 1-May 31	11	15,891	16,265	5,342	37,498
(max $\frac{H}{E}$ = 10.0)	12	10,547	17,744	2,968	31,259
	13	8,611	19,222	2,178	30,011
	14	7,550	20,701	1,785	30,036
	15	6,856	22,180	1,549	30,585
June 1-Oct. 10	23	43,208	73,593	2,133	118,934
(max $\frac{H}{E}$ = 21.8)	24	33,153	76,792	1,860	111,805
	25	28,945	79,992	1,731	110,668
	26	26,516	83,192	1,657	111,365
	27	24,887	86,391	1,609	112,887
Oct. 11-Dec. 14	17	154,162	26,785	3,461	184,408
(max $\frac{H}{E}$ = 16.5)	18	55,158	28,361	1,297	84,816
	19	35,357	29,936	867	66,160
	20	26,871	31,512	682	59,065
	21	22,156	33,088	581	55,825
	22	19,156	34,663	518	54,337
	23	17,079	36,239	474	53,792
	24	15,556	37,814	443	53,813
	25	14,391	39,390	419	54,200

Therefore, under these conditions Model I gives an overall grand total of 943,457.23 dollars and allows 281.40 acres to burn if firefighter positions are filled up to the required levels. To the nearest thousand dollars, this still means a saving of $4,022,000.00 when compared with the costs actually incurred.

Present Forest Service employment practices call for a firefighter to be on duty -- standby plus actual firefighting -- for 12 hours per day 6 days per week, or 72 hours per week. Thus to find the actual number of men

required to fill the necessary firefighter positions, it is necessary to calculate the total number of man-hours per week required for complete coverage and then divide this number by 72. This will give the number of men required per ranger district, which must then be multiplied by 6 to cover all districts on the forest. These calculations are shown in Table 3.8 where the last column shows the number of men needed to fight fires (exclusive of lookouts, patrolmen, etc.) on the Plumas National Forest during 1959. These numbers are clearly larger than the numbers employed under current practices, but it is seen that under the assumptions of Model I, the total costs can be considerably reduced by increasing the number of firefighters readily available for initial attack. As mentioned previously, this general conclusion is also supported by experimental evidence from the Increased Manning Experiments reported in [9].

Table 3.8
Number of Men Required

Period	Firefighter positions	Man-hours per week	Men per ranger district	Total number of men required
Apr. 1-May 31	13	1,092	16	96
June 1-Oct. 10	25	2,100	30	180
Oct. 11-Dec. 14	23	1,932	27	162

It is of interest to note the number of A, B, C, D and E fires occurring under the model calculations. Table 3.9 shows these figures for both man-caused and lightning-caused fires. Total acreage burned is 281.40 acres. Compare with Table 3.1 which gives the actual fire history.

Table 3.9

Number of Fires

Class	Theoretical number of man-caused fires	Theoretical number of lightning-caused fires	Total
Class A	44	71	115
Class B	13	6	19
Class C	3	1	4
Class D	1	0	1
Class E	0	0	0
Total	61	78	139

F. Consideration of Other Suppression Costs

One further calculation will be made here to make this comparison perhaps more realistic. Only manpower costs to control fires has been included in the theoretical suppression costs, while the actual costs include numerous other items, such as bulldozers, ground and airtankers, other aircraft, and certain other manpower costs. Heretofore it has been assumed that only manpower would be needed, but even if these other costs are added to the theoretical costs, the savings are still impressive. The actual suppression cost total of $754, 394 includes 29, 220 manhours spent to control the fires. Using the $4 per man-hour figure, this man-hour cost becomes $156, 880. Therefore, $597, 514 were spent for these other fire control items. If it is assumed that these costs will also be incurred under the model calculations, the total theoretical costs increase to $943, 457 plus $597, 514, or $1, 540, 971. This still means a saving, to the nearest thousand dollars, of $3, 424, 000 over the actual total costs incurred. Thus, even including all actual non-manpower suppression costs in the theoretical total, the savings predicted by the increased manning are considerable.

G. Comparison of Theoretical Savings

The theoretical versus actual cost comparisons discussed thus far in this chapter are summarized below in Table 3.10

Table 3.10

Theoretical Versus Actual Cost Comparisons

	Actual history	Unconstrained optimum solutions	Constrained optimum solutions	Constrained optimum plus other
Acres burned	11,477.75	217.32	281.40	281.40
Burned acreage costs	4,211,000.00	71,000.00	104,000.00	104,000
Suppression costs	754,000.00	28,000.00	839,000.00	1,437,000
Total Costs	4,965,000.00	99,000.00	943,000.00	1,541,000
SAVINGS		$4,866,000.00	$4,022,000.00	$3,424,000

The foregoing analyses assume that the distributions of growth rates and fire ignitions were known prior to the start of the fire season. Of course, this would not be completely possible, as only certain measures are currently available to estimate these distributions, such as cumulative rainfall, position of fuels in their growth cycles, historical weather patterns, and the like. Before optimal use of these models can be made for pre-fire season planning, it will be necessary to have adequate estimates of these distributions. However, within certain limits they can be predicted in advance, and manning procedures can be planned before the fire season. It is well to point out, however, that the problem of advance planning models for estimation of these distributions remains. It is strongly expected that even with these uncertainties the savings will still be significant.

H. Reduction of Attack Time

1. Introduction

At this point, a few words should be said about the possibility of reducing the attack interval, thereby increasing the transportation costs, but possibly reducing the overall total costs, even further. The U.S. Forest Service reports considerable success in the use of helicopters for initial attack transportation, which, while expensive, prevents certain fires from becoming large and therefore actually reduces total costs. If arriving at a fire sooner with a stronger attack force will save sufficient acreage from the fire, the extra expenses will be fully justified.

2. Assumptions

In order to investigate such possible economies, two of the 139 fires on the Plumas Forest in 1959 will be examined in some detail to see if use of a large helicopter could be justified by savings in acreage burned. The helicopter to be specifically considered here will be the Kaman H43B, or similar, which can carry eight passengers and has an average charter rate of $560 per hour. (Eight passengers could also be carried in four of the Bell 47 series helicopters, with approximately the same total hourly charter rate.) It is further assumed that helicopter standby is charged at the rate of two operating hours per day, that is, even if the helicopter is not used at all, a charge of $1,120 per day is incurred.

3. Calculation Procedure and Results

a. Medium Size Fire: The first fire to be considered is a relatively small Class C fire (identified by the Forest Supervisor's fire number 126) which actually burned a total of 13 acres, 7 in resource Class 4 and then 6 in resource Class 5, for a total damage figure of $14,250. Actual suppression costs were reported as $1,625, making the total cost of this fire

\$15,875. This fire was four acres in size when discovered and was attacked by two men after 1.10 hours. The distance to be traveled was three miles. Assume that the attack could have been made in 15 minutes by helicopter, so that the round trip travel time is 0.50 hours. Since eight men can be transported in one trip, the per man transportation cost is \$560 times .50 hours divided by eight, or \$35 per man. From the fire records the fire parameters are given by:

$G_A = 10.67$

$H = 7.11$

$E = 4.20$

$C_B = 750$

$C_S = 35$

$C_X = 4$

$C_F = C_T = 0$.

Solving Equation (2.18) gives an optimum additional number of men, z^*, of 17.07 . Adding H/E , the fixed base of suppression forces, to this, yields a total optimum suppression force, x^*, of 18.76. This would indicate that at least two trips by the helicopter are required, the second group of men to be reinforcements. However, it is shown in [3] that if the reinforcement interval is greater than G_A/Ez_1 , where z_1 is the number of men sent initially, no reinforcements should be sent. In this case, the reinforcement interval is 1/2 hour (the round trip travel time), and since $z_1 = 8.00 - H/E = 6.31$, G_A/Ez_1 is equal to .40 hours. Hence, the best solution is to send eight men on initial attack and to send no reinforcements; that is, only one trip by the helicopter is required.

From (2.7) the controllable area of burn is equal to 2.15 acres, and from (2.3) and (2.4), the size of the fire at time of attack is 6.44 acres,

giving a total area burned by this fire of 8.59 acres. The control time is given by (2.10) as .40 hours. Then from (2.13)

$$C = 0 + 35(8) + 4(.40)(8) + 750(7) + 1,500(8.59 - 7)$$
$$= \$7,928 \quad .$$

Use of the helicopter thus provides a saving of \$7,947 over the actual total costs. Since the helicopter costs a minimum of \$1,120 per day for standby, its use could be justified for approximately 7 days on standby <u>for fire use only</u> by the savings on this one relatively small fire alone.

b. Large Fires: If an even larger fire could be prevented by faster initial attack, the realized savings are more spectacular. Consider the actual fire, identified by the Supervisor's number 69, which burned a total of 1,450 acres, 1,440 in resource Class 3 and 10 in resource Class 2, for a total damage cost of \$505,250. Actual suppression costs were reported as \$58,000, giving a total actual fire cost of \$563,250. Initial attack on this fire was made by three men traveling 23 miles in 0.42 hours. Assume that helicopter attack could be made in 20 minutes, or a round trip time of 0.67 hours. The transportation cost then becomes \$46.67 per man. Examination of fire records yields the following parameter values:

$$G_A = 6.22$$
$$H = 0$$
$$E = 2.04$$
$$y_A = 0$$
$$C_B = 350$$
$$C_S = 46.67$$
$$C_X = 4$$
$$C_F = C_T = 0 \quad .$$

Calculations yield $x^* = 8.46$ men, or one trip by the helicopter, and a final area burned of 1.19 acres. The theoretical total costs are thus:

$$C = 0 + 46.67(8) + 4(.38)(8) + 350(1.19)$$
$$= \$802 \quad .$$

This means a theoretical savings of $562,450 over actual costs, or, it justifies the helicopter on standby for approximately 502 days on the savings from this fire alone. Since the fire season on the Plumas in 1959 (from April 1 to December 14) was 258 days long, the savings from this one fire would justify helicopter standby for almost two entire fire years. It should be emphasized that the helicopter would be justified by fire control use only, and any considerations of multiple use would only increase its justification.

Clearly some of these analyses indicate fundamental adjustments of fire planning policies are in order; the potential savings of these changes cannot be ignored. The final section will examine this question, as well as indicate various areas where further research is needed to improve the model structures and make the overall analyses directly useful for fire planning and budgeting.

4. SUMMARY AND AREAS FOR FURTHER RESEARCH

A. Implications for Planning Policy

To use the concepts developed in the previous chapters to obtain the predicted savings will require some basic changes in current planning and budgetary procedures. First, the objective of least total cost must be generally accepted; damages caused by fire must be recognized as being as much an out-of-pocket cost as expenditures for suppression. Damage costs on National Forest lands are a real cost to the public, and considering them as such makes the least cost approach not only reasonable but mandatory. Any other approach is, from a long range view, suboptimal.

The principal policy change required to operate under these concepts can be stated in one sentence: Spending more money in advance of a fire for presuppression activities and increased initial attack capabilities will greatly decrease the area burned by fire and also the total costs of fire.

This is hardly an unexpected conclusion, as it has already been indicated experimentally[9] but for various reasons it is one which has not been agreed to nor satisfactorily acted upon. The magnitude of the funds that should be available for pre-fire use may come as a surpise to some, but if least total cost is accepted as a desirable objective, the expenditure of these funds may be recognized as a necessity. Previously this conclusion could be based only on opinion, which, however well founded, was not entirely satisfactory. It is hoped that this work will provide a preliminary step towards quantitative economic justification for increasing budgetary expenditures for pre-fire activities.

The major change in planning policy indicated by the foregoing analysis is to budget and spend not only the Presuppression and Management (P and M funds) but also more of the money which in general is now only available after a fire escapes initial attack (FFF funds) in advance of the fire starts for additional initial attack capabilities; that is, to greatly increase the operating budgets of the National Forest.

Table 4.1 gives the actual cost break down for 1959 on the Plumas National Forest. Compare with Table 3.10 to see the magnitude of the changes required.

Table 4.1

Actual Fire Costs for 1959, Plumas National Forest

SOURCE	AMOUNT (dollars)	
P and M:		
Presuppression and Prevention	137,343	
Equipment Use	19,546	
Supplies	30,867	
Other	2,363	
Subtotal		190,119
FFF:		
Presuppression and Prevention	118,560	
Suppression	735,911	
Subtotal		854,471
State Funds:		
Presuppression, Prevention and Suppression	57,264	
Equipment Use	5,282	
Supplies	7,882	
Other	165	
Subtotal		70,593
TOTAL		1,115,183

B. Problem of Resource Valuation

Least total cost analysis of the forest fire suppression problem requires a rational, realistic system for valuation of the resources being protected. Actually attaching a dollar value to these resources is a long-standing problem among those concerned with wildland management. An excellent start toward quantification of the many resource values has been made on the National Forests under the instructions given in the Forest Service Handbook [8], and it is these values that were used in the analysis in Section 3. However, it would appear that these values are not currently in general use by fire control planners to measure fire-caused damages. If these valuations are really to be believed, then their rational use in planning is mandatory; if they are not to be believed, then the justification for their existence is questionable. General acceptance of the least total cost objective and some reasonable system of resource valuation should enable great reductions to be made in wildland fire costs.

Although the least cost analyses presented in the preceding chapters obviously depend on the values given to the wildland (C_B measures these values), the conclusions thus derived are not negated by considering lower values than those obtained by following the procedures in the Forest Service Handbook. In fact, if current planning were done by using the models presented earlier, the C_B values implied by the resultant action are considerably lower than would be given by the most skeptical observer. To see this, consider the two actual fires that were analyzed in some detail in Section 3.1. The models suggested in the previous chapters all result in x^*, the optimal size of the suppression forces, being proportional to the square root of the C_B values. Thus, it can be said

$$(4.1) \qquad \left(\frac{x'}{x}\right)^2 = \frac{C'_B}{C_B}$$

where the primed variables are what is implied by current planning and the non-primed ones are those used in these models. Thus, for the first fire of Section 3.1 (Supervisor's fire number 126), x' equals two, the actual number of men sent on initial attack; x equals eight, the optimal number to send when using helicopter initial attack; and C_B equals 750 dollars/acre, the value used in the optimization by the model. Solving (4.1) for C'_B, the resource valuation implied by the Forest Service actions, gives a value of approximately 47 dollars/acre. It must be generally agreed that fire causes more damage than indicated by this figure. A similar calculation for the Supervisor's fire number 69 gives a C'_B of 49 dollars/acre when C_B is 350 dollars/acre.

The important conclusion here is that even if the C_B figures used previously are not generally accepted, increased initial attack capabilities are still required under these models, since the currently implied figures are much too low to be reasonable. Thus, changes in planning policies are seen to be required even without accurate valuation procedures, but maximum benefits will not be obtained from least cost analysis until a consistent, rational system of resource valuation is generally accepted and applied. Research in this area is urgently needed.

C. Data Collection

Although these models could be used on a going fire by making on-the-spot estimates of the various fire parameters, their real usefulness comes in the area of pre-fire planning and a priori manning for adequate initial attack capabilities. This means that estimates for the parameters

must be obtained for the wide variety of fuel and weather conditions that may be encountered in the field. Past fire records are useful for obtaining rough estimates of these parameters under various conditions (see [4] Appendix E), but in most cases the information available is too limited to permit precise estimation.

Experimentation is perhaps the best way to obtain these estimates, although it is time-consuming and not without risk. To illustrate the difficulty, it might be necessary to set free-burning test fires under different field conditions and record the size of the fire, say, every five minutes, and then to attack the fires with different types of suppression forces and keep track of the progress toward control of the fire. As far as the fire growth parameters are concerned, some experiments of this type have already been done,[11],[17] but in [11] the fires were too small and the weather conditions too favorable for the data to be particularly useful, and in [17] only a few highly specialized fuel types were considered and not enough weather parameters were measured. These experiments were on the right track, but they need to be greatly expanded in scope. To this author's knowledge, there is virtually no experimental data, other than that on fireline construction, on the second type of information needed, that of the effect of different types of suppression activity on fires of various physical characteristics.

Much of this data will not be available in the immediate future, but in the meanwhile trained observers on actual fires could do a great deal in developing this information. Procedures for collecting such data could be readily developed, responsibility assigned, and the data collection work could begin immediately. Recent advances in infra-red photography techniques may be very helpful in this area.

D. Model Research

The analytical model presented here contains certain simplifications, and some aspects of the fire control problem are omitted entirely. As indicated in section 1, this does not make the results thus obtained worthless, but it does suggest areas of further research to make these models more usefulin actual decision-making.

Some of these areas are discussed in [3], but theoretical work is still needed on many aspects of fire modelling, including prevention, detection, attack and control, mop-up and demobilization, and the special problems of large fires. (See [6])

E. In Conclusion

This paper is intended to present the beginnings of an analytical study of not only the problem of initial attack and control but other problems of forest fire research as well.

The conclusions drawn from this work and the implications for future policy decisions, although not totally unexpected, should help to provide a sounder basis for fire control planning than exists at present. The transformation of physical and operational concepts into engineering terms is not an easy one, and certainly this paper does not provide either the final word in fire spread model building or immediately usable decision-making tools for the front-line fire control officer. It does provide, however, a rational basis for long-range planning and the start of a sound economic basis upon which to justify increased expenditures for the strengthening of fire control organizations and agencies to meet present-day requirements. Finally, it hopefully will also provide inspiration to others to continue operations research studies of this extremely important subject area.

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